Supplementary information

Wim A. van Duyl[†]*

[†]Formerly at Department of Medical Physics and Technology, Erasmus University Rotterdam, 50 3062 PA Rotterdam and [†]Department of Electronic Instrumentation, Delft Technical University, 5 2600 AA Delft, The Netherlands

*Corresponding author: Wim A. van Duyl, Email: wavanduyl@cs.com

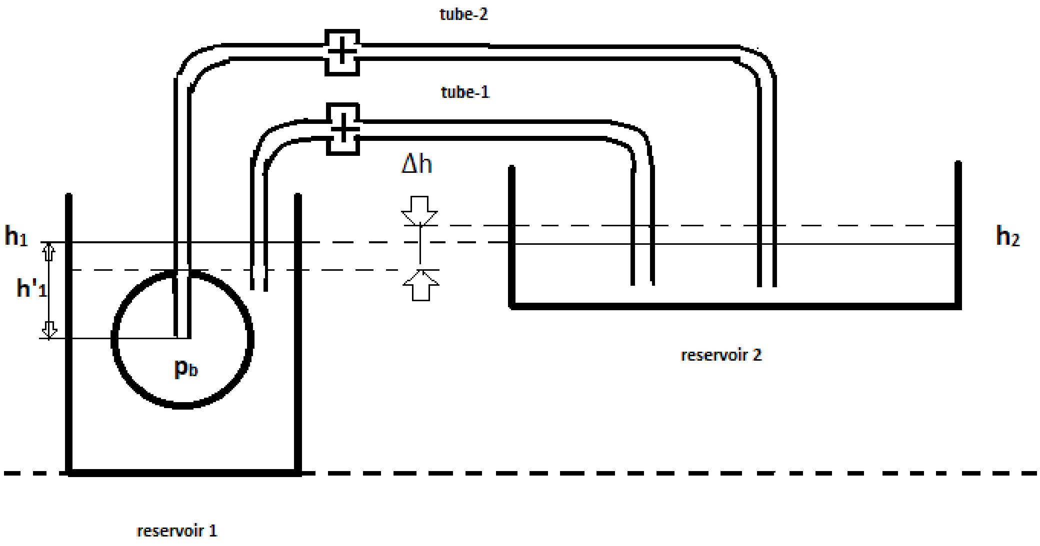

Figure S1. Two reservoirs connected by tube 1 which if opened brings water levels in both reservoirs in hydrostatic equilibrium. One end of tube 2 is situated in an elastic balloon and the other end is situated in reservoir 2. If tube 1 is closed and tube 2 is open, the stress in the wall of the balloon causes a flow out of the balloon via tube 2 into reservoir 2.

Figure S1 represents reservoir 1 and 2, both filled with water and connected to each other via tube 1 that is closed but can be opened. If there is a difference between the water levels in both reservoirs Δh then this causes a hydrostatic pressure difference $\Delta p_h = \Delta h = h_1 - h_2$ across this tube. If tube 1 is opened water flows from the reservoir with the highest water level to the reservoir with the lower water level till the water levels in both reservoirs are equal and $\Delta h(0) = 0$. In the initial situation drawn in Figure 1A $\Delta h(0) = 0$. In reservoir 1 an elastic balloon filled with water is fixed in the water. Total volume of the balloon is V that is equal to the sum of rest volume V_R and elastic volume V_E. Rest volume V_R is assumed to be constant. Elastic volume V_E depends on pressure p_b in the balloon. The balloon is connected to reservoir 2 via tube 2 that is closed but can be opened. Pressure inside the balloon p_b is measured at a fixed point h_1' under water level. Stress in the wall of the balloon contributes a pressure p_d to p_b. In the condition that $p_d = 0$ and $\Delta h(0) = 0$, p_b is calibrated as $p_b = 0$. When tube 2 is opened while $p_d(0) > \Delta p_h(0)$ then there is a flow F from reservoir 1 to reservoir 2 and Δp_h increases. This flow stops when $p_d(final) = \Delta p_h(final)$. For flow F via tube 2 from reservoir 1 to reservoir 2 holds,

$$p_d = FR + \Delta p_h \tag{1A}$$

where p_d is the flow driving pressure that is built up by stress in the wall of the balloon and R is flow resistance of tube 2.
Cross sectional area of reservoir 1 is $A_1=125\ cm^2$and cross sectional area of reservoir 2 is $A_2=625\ cm^2$. When time t after initial condition $\Delta h(0) = 0$ a certain volume $V_e(t)$ has streamed from reservoir 1 to reservoir 2, then the water level in the reservoir 1 decreases with Δh_1 and water level in reservoir 2 increases with Δh_2 such that $\Delta h_1(t) A_1 = \Delta h_2(t) A_2 = V_e(t)$. The expelled fluid is accompanied by a change in hydrostatic pressure difference $\Delta p_h(t)$:

$$\Delta p_h(t) = (1/A_1 + 1/A_2)V_e(t) \tag{2A}$$

As $A_2>A_1$ the decrease of water level h_1 is larger than the increase of level h_2.

Because changing column h_1' above the point of measurement contributes to the change Δp_b the recorded change in pressure Δp_b is not equal to the change in Δp_d. Because $\Delta h_1=\Delta h'_1$,

$$\Delta p_b(t) = \Delta p_d(t) - \Delta h'_1(t)= \Delta p_d(t) – V_e(t)/A_1 \tag{3A}$$

The finally expulsed volume $V_e(final)$ is accompanied by final decrease of p_d from $p_d(0)$ till $p_d(final)$:

$$\Delta p_h(final)= (1/A_1 + 1/A_2)V_e(final) \tag{4A}$$

For ratio $V_e(final)/\Delta p_d(final)$ we write:

$$V_e(final)/\Delta p_d(final) = V_e(final)/(p_d(0) - p_d(final))= V_e(final)/[p_d(0) – V_e(final)(A_1+A_2)/A_1A_2]=C \tag{5A}$$

The ratio C of a small change of elastic volume dV_E and associated small change in pressure dp_d of an elastic balloon is the elastic compliance C_E:

$$C_E = dV_E/dp_d \tag{6A}$$

We assume that for the elastic balloon C_E is constant, independent of V_E. In other words for such an elastic balloon $C_E=C$ so that $V_E/p_d(0) = C_E= C$ and according to (5A) C_E can be derived from the observed values of $V_e(final)$ and $p_d(0)$.

We express the change in hydrostatic pressure caused by change in h_1 and h_2 also in a compliance parameter C_h defined as:

$$C_h = dV_e/dp_h= A_1A_2/(A_1 + A_2)=V_e(t) /\Delta p_h(t) \tag{7A}$$

In the set up C_h is a constant. With the given values of A_1 and A_2, C_h = 104 ml/cm H_2O.
By combining (7A) with (5A) we get:

$$C= C_E= V_e(final)/\Delta p_d(final) = V_e(final)/[p_d(0) – V_e(final)/C_h] \tag{8A}$$

When tube 1 is closed then F is equal to rate of decrease of the elastic volume $V_E(t)$ so that:

$F(t) = -dV_E(t)/dt$ (9A)

The combination of (6A) with (9A) yields

$F(t) = -C_E\, dp_d/dt$ (10A)

The combination of (10A) with (1A) yields

$p_d(t) = -RC_E\, dp_d/dt + \Delta p_h(t)$ (11A)

Combined with (7A) we can rewrite (10A) as:

$p_d(t) = -RC_E\, dp_d/dt + V_e(t)/C_h$

or

$dp_d/dt = -(C_h\, p_d(t) - V_e(t))/RC_EC_h$ (12A)

With initial elastic volume of the balloon $V_E(0)$ and constant elastic compliance C_E holds:

$V_e(t) = V_E(0) - p_d(t)C_E$ (13A)

By substitution of (12A) in (13A) we get the following first order differential equation:

$$\frac{dp_d}{dt} = -\left[\frac{C_E + C_h}{RC_EC_h}\right]p_d(t) + \frac{V_E(0)}{RC_EC_h} \qquad (14A)$$

With initial value $p_d(0) = V_E(0)/C_E$ the solution of differential equation (14A) is

$$p_d(t) = V_E(0)\frac{C_h}{C_E(C_E+C_h)}e^{-\frac{t}{RC_s}} + \frac{V_E(0)}{C_E + C_h} \qquad (15A)$$

where for C_s in the exponent time constant $T=RC_s$ holds

$C_s = C_EC_h/(C_E + C_h) < C_E$ (16A)

C_s is an equivalent compliance parameter of the system that determines the rate of elastic expulsion and C_s equals the value of C_E in series with C_h,.

According to (15A) $p_d(t)$ decays according to a mono-exponential function with time constant $T = RC_s$.

Besides by using (8A) we can use (16A) also to derive the elastic compliance C_E from the time constant T obtained from the mono-exponential function:

$C_E=C_hC_s/(C_h - C_s)$ (17A)

The hydrostatic pressure Δp_h built up during expulsion has an accelerating effect on decay of $p_d(t)$ so that $C_s < C_E$.

In the experimental set up $p_b(t)$ has been recorded instead of $p_d(t)$.

During an expulsion h'_1 decreases similar in time to the increase of $V_e(t) = C_E\, p_d(0) - C_E\, p_d(t)$. If we use (3A) to substitute $p_b(t)$ for $p_d(t)$ we get,

$p_b(t)= (1+C_E/A_1]\ p_d(t)- C_E/A_1\ p_d(0)$ (18A)

so we find that $p_b(t)$ decreases similar to the mono-exponential decay of $p_d(t)$, with time constant $T= RC_s$. Hence time constant T derived from recording of $p_b(t)$ we can calculate, by applying (15A), (16A) and (17A) with known values of R and C_h , the elastic compliance C_E .

The amplitude $p_d(0)$ can be derived from the amplitude of $p_b(0)$ by applying:

$P_d(0) = p_b(0) + \Delta p_h(final) = p_b(0) + V_e(final)/C_h$ (19A)

Because:

$V_e(final)= C_E\ [p_d(0) - p_d(final)]=C_E[p_b(0) – (p_b(final)-\Delta h_1)]=$

$C_E[p_b(0) – (p_b(final)-V_e(final)/A_1)]$

or

$V_e(final)\ (1+1/A_1)= C_E(p_b(0) – p_b(final))$ (20A)()

the total amount of expelled volume $V_e(final)$ expelled by the mono-exponentially decaying pressure $p_d(t)$ can be derived from the mono-exponentially decaying $p_b(t)$.